Enrollment No: ______________________ Exam Seat No: ______________________

C. U. SHAH UNIVERSITY

Winter Examination-2022

Subject Name: Mathematical Methods-I

Subject Code: 5SC03MAM1 **Branch: M.Sc. (Mathematics)**

Semester: 3 **Date: 22/11/2022** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the following questions** **(07)**

a. State Parseval's formula for Fourier series. **02**

b. If $F_s[f(x)] = F(\lambda)$, then prove that **02**

$$F_s[f(x)\cos ax] = \frac{1}{2}[F_s(\lambda + a) + F_s(\lambda - a)].$$

c. Write Dirichle's conditions for Fourier series. **02**

d. $f(x) = 1, 0 < x < \infty$cannot be represented by a Fourier integral. (True or False) **01**

Q-2 **Attempt all questions** **(14)**

A. Find the Fourier series of the function $f(x) = x$ in the interval$(0, 2\pi)$. **05**

B. Determine the Fourier series expansion of the function **05**

$f(x) = \begin{cases} -k, & -\pi < x < 0 \\ k, & 0 < x < \pi \end{cases}$ where $k > 0$. Deduce that

$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} = \frac{\pi}{4}$.

C. Obtain the half-range sine series for the function $f(x) = x^2$ in the interval $0 < x < 3$. **04**

OR

Q-2 **Attempt all questions** **(14)**

A. Determine the Fourier series expansion of the function$f(x) = |x|$ in the interval $-\pi < x < \pi$. Deduce that $1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$. **05**

B. Obtain the Fourier series for the function $f(x) = x - x^2$ in the interval $-1 < x < 1$. **05**

C. Find the half-range cosine series of the function **04**

$$f(x) = \begin{cases} x, & 0 < x < \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} < x < \pi \end{cases}.$$

Q-3 **Attempt all questions** **(14)**

A. State and prove Convolution Theorem of Fourier transform. **06**

B. If $F(\lambda)$ is the Fourier transform of $f(x)$, then show that **05**

$(\boldsymbol{i})\ F[f(ax)] = \frac{1}{a}F\left(\frac{\lambda}{a}\right), a \neq 0, (\boldsymbol{ii})\ F[f(x-a)] = e^{i\lambda x}\ F(\lambda).$

C. Find the Fourier sine integral representation of $f(x) = \begin{cases} 0, & 0 < x < 1 \\ k, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$ **03**

where k is constant.

OR

Q-3 **Attempt all questions**

A. Let $u(x,t)$ be a function of two variables and let $u(x,t)$ and $u_x(x,t)$ both tend to 0 as $x \to \infty$. Then show that **06**

$(\boldsymbol{i}) F_c[u_x](\lambda) = -\sqrt{\frac{2}{\pi}}u(0,t) + \lambda F_s[u](\lambda),$

$(\boldsymbol{ii}) F_c[u_{xx}](\lambda) = -\sqrt{\frac{2}{\pi}}u_x(0,t) - \lambda^2 F_c[u](\lambda).$

B. Using Fourier integral representation, prove that **05**

$$\int_0^\infty \frac{\cos\lambda x + \lambda\sin\lambda x}{1+\lambda^2}\, d\lambda = \begin{cases} 0, & x < 0 \\ \frac{\pi}{2}, & x = 0 \\ \pi e^{-x}, & x > 0 \end{cases}.$$

C. Find the Fourier sine transform of $f(x)$, if $f(x) = \begin{cases} 0, & 0 < x < a \\ x, & a \le x \le b \\ 0, & x > b \end{cases}.$ **03**

SECTION – II

Q-4 **Attempt the following questions** **(07)**

a. If $L[f(t)] = \bar{f}(s)$, then show that **02**

$$L[\cosh at \cdot f(t)] = \frac{1}{2}\left[\bar{f}(s-a) + \bar{f}(s+a)\right].$$

b. Find the inverse Laplace transform of $\bar{f}(s) = \frac{s^2-3s+4}{s^3}$. **02**

c. Compute the Z-transform of $\left\{\frac{a^n}{n!}\right\}_{n\ge 0}$. **02**

d. Z-transform of unit impulse sequence $\delta(n) = \begin{cases} 1, & k = 0 \\ 0, & k \neq 0 \end{cases}$ is 1. **01**

(True or False)

Q-5 **Attempt all questions** **(14)**

A. Using the Laplace transform, solve the IBVP described as **07**

PDE: $u_{tt} = u_{xx}, 0 < x < 1, t > 0$

BCs: $u(0,t) = u(1,t) = 0, t > 0$

ICs: $u(x,0) = \sin\pi x, u_t(x,0) = -\sin\pi x, 0 < x < 1.$

B. Apply the convolution theorem to evaluate $L^{-1}\left[\frac{s}{(s+a)(s^2+1)}\right]$. **04**

C. If $L[f(t)] = \bar{f}(s)$, then show that **03**

$$L[f'''(t)] = s^3\bar{f}(s) - s^2 f(0) - sf'(0) - f''(0).$$

OR

Q-5 **Attempt all questions**

A. Solve $y'' + 9y = \cos 2t$ subject to $y(0) = y'(0) = 1$ using Laplace transform. **04**

B. If $f(t)$ is a periodic function with period T, then prove that **04**

$$L[f(t)] = \frac{1}{1-e^{-sT}} \int_0^T f(t) e^{-st}\ dt.$$

C. Compute the Laplace transform of the function **03**

$$f(t) = \begin{cases} \sin t, & 0 < t < \pi \\ 0, & t > 0 \end{cases}.$$

D. Find the inverse Laplace transform of $\bar{f}(s) = \cot^{-1}\left(\frac{s}{k}\right)$. **03**

Q-6 **Attempt all questions** **(14)**

A. Show that $Z[\cosh n\theta] = \frac{z^2 - z\cosh\theta}{z^2 - 2z\cosh\theta + 1}$. **04**

B. Compute the inverse Z-transform of $U(z) = \frac{z}{(z-2)(z-3)}, |z| > 3$. **04**

C. Construct Green's function for the boundary value problem $y'' = x^2$ subject to $y(0) = y(1) = 0$. **03**

D. Obtain the inverse Laplace transform of $\bar{f}(s) = \frac{\pi e^{-s} + s e^{-\frac{s}{2}}}{s^2 + \pi^2}$. **03**

OR

Q-6 **Attempt all Questions**

A. Obtain $Z^{-1}\left\{\frac{z^3}{(z-3)(z-2)^2}\right\}, |z| > 3$. **05**

B. State Gram-Schmidt orthonormalization process and hence othonormalize the set $\{1, x, x^2\}$ over $[-1, 1]$. **05**

C. If $Z[u_n] = U(z)$, then show that $Z[nu_n] = -z\frac{d}{dz}U(z)$. **02**

D. Prove that $L[t \sin at] = \frac{2as}{(s^2+a^2)^2}$. **02**